ISSN 1066-369X, Russian Mathematics, 2017, Vol. 61, No. 8, pp. 36–45. © Allerton Press, Inc., 2017.
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Uniformization of One-Parametric Families of Complex Tori

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Received April 7, 2016

Abstract—We suggest an approximate method to find an elliptic function uniformizing a compact Riemann surface of genus 1 which is given as a ramified covering of the Riemann sphere. The method is based on including the surface into a smooth one-parametric family. We deduce a system of ordinary differential equations for critical points of elliptic functions uniformizing surfaces of the family.

DOI: 10.3103/S1066369X17080047

Keywords: *Riemann surface, complex torus, ramified covering, elliptic function, uniformization, one-parametric families of meromorphic functions.*

INTRODUCTION

The parametric method is one of the main tools in the theory of univalent functions. It was suggested by Loewner and developed by many his successors, including Kufarev (see, e.g., [1–3]). The essence of the parametric method is in including of a univalent in the unit disk D function $f(z)$ into a family $f(z,t)$, $t \geq 0$, as an initial element ($f(z,0) = f(z)$) and investigation of the differential equation

$$\dot{f}(z,t) = zf'(z,t)h(z,t), \quad z \in D, \ t \geq 0,$$

which the family satisfies to. With the help of parametric method, many difficult extremal problems were solved, in particular, it was proved the famous Bieberbach conjecture on estimates of coefficients of univalent in the unit disk functions [4]. There are also known analogs of the Loewner equation for other domains, for example, for the half-plane (chordal equation) and for an annulus [1]. In recent years, interest increased to stochastic Loewner equation or Schramm–Loewner equation (SLE) (see, e.g., [5–8].).

For investigation of non-univalent functions, such as polynomials and rational functions, it is important to consider one-parametric families of multivalent functions of meromorphic functions mapping the Riemann sphere $\overline{\mathbb{C}}$ or its subdomain G onto Riemann surfaces with branch-points, projections of which depend on the parameter.

In our paper [18] we consider one-parametric families of polynomials, mapping $\overline{\mathbb{C}}$ onto Riemann surfaces over $\overline{\mathbb{C}}$, and deduce a system of ODEs to determine critical points of a polynomial uniformizing a given Riemann surface. Here we solve a similar problem for complex tori over $\overline{\mathbb{C}}$ with simple finite branch-points; these results were announced in [10, 11]. In addition, we deduce an analog of the Loewner equation for multivalent functions.

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